ON THE DESIGN OF RHOMBIC ANTENNAS

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ON THE DESIGN OF RHOMBIC ANTENNAS

by

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PREFACE

Early in 1952, J. G. Chaney, working at the United States Naval Postgraduate School, integrated the generalized circuit of the horizontal free-space rhombic antenna and obtained an expression for its radiation impedance in terms of tabulated functions. This expression and the results of an earlier paper, by the same author, on the application of circuit concepts to field problems have enabled this writer to present a design procedure for horizontal rhombic antennas which is based, to a great extent, on generalized circuit theory and which gives results which agree well with the recent experimental work of Christiansen.

My thanks are due to "The Manager, RCA Review" and E. A. Laport for permission to copy Table II and Figure 4, and to McGraw-Hill Book Company for permission to copy Figures 2 and 3. I wish also to express my gratitude for the kind assistance of Professors J. G. Chaney and C. F. Klamm, Jr., of the U. S. Naval Postgraduate School.

Neil I Heenan

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TABLE OF SYMBOLS AND ABBREVIATIONS

ℓ	The side length of the rhombic antenna
h	The distance between the horizontal rhombic and its image
Δ	The 'Angle-of-fire' of the transmitting rhombic
λ	The wavelength
$2\underline{a} = A$	The aperture or acute angle of the rhombic
n	$\ell = \frac{n\lambda}{4}$ where n is ≥ 8
r	Radius of the wire forming the active element of the rhombic
λ^m	The wavelength in meters
d	The diameter in inches of the wire forming the active element of the rhombic
Z_r	The free-space radiation impedance
R_r	The free-space radiation resistance
X_r	The free-space radiation reactance
$Z_o = R_o + jX_o$	The terminating impedance of the Rhombic Antenna
l_n	The natural logarithm
log	The common logarithm
α	The attenuation factor in nepers **pers meter**
$Z_{in} = R_{in} + j X_{in}$	The Driving point impedance
T	The db terminal loss
g_d	The directivity
G_d	The power gain for $f_m = 1$
G_p	The power gain above ground

$G_{p\frac{\lambda}{2}}$ The power gain with reference to a horizontal half wave dipole in the same position

$\beta = \frac{n\pi}{4}$ $(1 - \cos \Delta \cdot \cos \underline{a})$

K The radiation intensity in watts per steradian

$|K|$ The time peak value of radiation intensity

$k = \frac{2\pi}{\lambda} = w(\mu_o \epsilon_o)^{1/2}$

ϕ The angle of azimuth

θ The angle from zenith

δ The complement of θ (Δ is a particular δ)

$\beta_1 = \frac{k\ell}{2}(1 - \cos \psi_1)$, $\cos \psi_1 = \sin\theta \cos(\phi + \underline{a})$

$\beta_2 = \frac{k\ell}{2}(1 - \cos \psi_2)$, $\cos \psi_2 = \sin\theta \cos(\phi - \underline{a})$

I_o The time peak driving point current in amperes

C Euler's constant (0.577216)

$C_i(x)$ The cosine integral of $x = -\int_x^\infty \frac{\cos u}{u} du$

$S_i(x)$ The sine integral of $x = \int_0^x \frac{\sin u}{u} du$

Z_i The internal impedance of the circuit in ohms per loop meter

f_m^2 The mean square current modulus (i.e. The mean square magnitude of the normalized current distribution along the antenna)

f_o^2 The attenuation factor at the terminating impedance Z_o

$\oint_1$ The contour integral along the axis of the wires forming the antenna

$\oint_2$ The contour integral along the inner periphery of the wires forming the antenna

$I_o f(P)$ The current at any point P in the circuit in terms of the current at an arbitrary reference point P_o. This reference point usually being taken at the point of maximum current or at the driving point.

P_1 Any point along the axis of the wire

P_2 Any point along the inner periphery of the wire

$(\ )^*$ The complex conjugate. Used also to indicate a footnote.

$\bar{\bar{\nabla}}_1 = \nabla_1 \nabla_1 + k^2 =$ The steady-state form of the differential operator deltil with the subscript indicating the position at which the differentiations are to be performed

$\underline{r}_{12}$ The vector distance from P_2 to P_1

r_{12} The distance between P_2 and P_1

$e(r_{12}) = r_{12}^{-1} \exp(-jkr_{12})$

$\mu_o = 4\pi \times 10^{-7}$ henries per meter.

$\epsilon_o = (36\pi \times 10^9)^{-1}$ farads per meter

W_o = The input power

ON THE DESIGN OF HORIZONTAL RHOMBIC ANTENNAS

I. Introduction

Summary: Utilizing generalized circuit theory wherever possible, a design procedure is laid down for the single wire, uni-element horizontal rhombic antenna, whose side length is an integral number of quarter wavelengths at the lowest design frequency, suspended above a perfectly conducting ground plane. A brief statement of the design procedure is followed by a discussion of the steps outlined. Following this some graphical design aids are introduced. Finally some computed values are compared with the experimental results of Christiansen.

In the design of Rhombic Antennas there are usually many variables. It is customary, therefore, to design, first of all, for assumed idealized conditions and then to consider the effects of the neglected variables. In general, a theoretical design is followed by experimental modification. It is the purpose of this paper to lay down the theoretical design considerations for the uni-element horizontal rhombic antenna at a height h/2 above a perfectly conducting ground-plane.

II. The Design Procedure

For the antenna shown in figure 1 the design procedure can be laid down as follows:

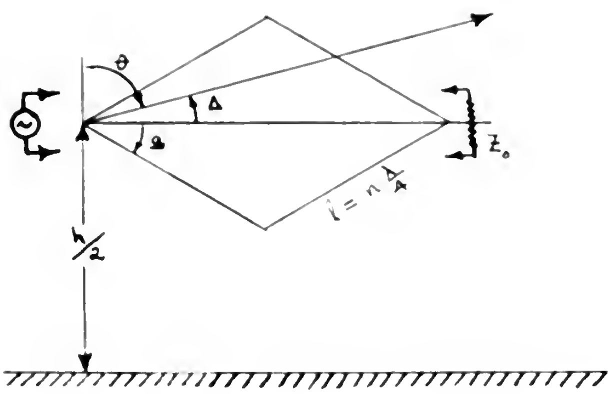


Figure 1

(a) Select $\ell > 2\lambda$ for the lowest frequency to be used.

(b) Choose the desired vertical angle of radiation Δ at the highest frequency to be used.

(c) If other engineering considerations permit, pick the height $h/2$ to yield maximum reinforcement in Δ . The height factor is a maximum when $\mathrm{Sin}\left(\frac{kh}{2}\,\mathrm{Sin}\,\Delta\right) = 1$, $h = \frac{\lambda}{2}\,\mathrm{cosec}\,\Delta$, where h is the antenna height above its image.

(d) The polarization of the radiated fields of the rhombic antenna are horizontal only in the $\phi = 0$ plane and in the $\theta = \frac{\pi}{2}$ plane. It is customary, therefore, to orientate the rhombus such that its principal diagonal is in the direction in which the signal is to be beamed.

Of all possible designs at least three have been named by previous authors. In one, the maximum of the principal lobe is obtained at an angle Δ . This is called an " alignment design". In the "maximum field strength" design the maximum field strength is obtained for a fixed ℓ , in the desired direction Δ although the directional maximum does not occur at the angle Δ .

In the "maximum output design", of Bruce, Beck and Lowry[3], the values of antenna height, side element length and aperture angle are prescribed to give maximum radiation intensity at an angle Δ for a fixed value of the current.

In the "alignment design" procedure the angle $\underline{a}$ is determined from the relationship

$$\mathrm{Cos}\,\underline{a} = \frac{n - 1.4852}{n \cos \Delta}$$

For a "maximum field strength" design with a given side length ℓ, $\underline{a}$ is determined as follows:

First, for the given Δ determine $\underline{a}$ to satisfy the condition

$$2\beta \cot\beta = \frac{\cos\Delta - \cos\underline{a}}{\sin^2\underline{a}\ \cos\Delta} \quad \text{where } \beta = \frac{kl}{2}(1-\cos\Delta\cos\underline{a})$$

$$= \frac{n\pi}{4}\ (1 - \cos\Delta\ \cos\underline{a}\ ).$$

With this value of $\underline{a}$ determine R_o as described in section (e).

With this value of R_o recompute $\underline{a}$ to satisfy the relation:

$$2\beta\cot\beta = \frac{\cos\Delta - \cos\underline{a}}{\sin^2\underline{a}\cos\Delta} + \frac{60}{R_o}\left[\frac{\cos\underline{a} - \cos\Delta}{\cos\Delta\ \sin^2\underline{a}} + 1\right]$$

The values of $\underline{a}$ and ℓ for a maximum output design are given by the relations

$$\frac{60}{R_o - 60} = \frac{\cos\underline{a} - \cos\Delta}{\sin^2\underline{a}\ \cos\Delta}\ ; \quad \ell = \frac{\lambda}{2(1-\cos\Delta\ \cos\underline{a})}$$

By varying side element length it is possible to obtain a maximum output alignment design. In this case

$$\frac{R_o - 60}{R_o} = \frac{\cos\Delta}{\cos\underline{a}} \cdot \frac{1 - \cos^2\underline{a}}{1 - \cos\Delta\ \cos\underline{a}}\ ; \quad \ell = \frac{.371\ \lambda}{1 - \cos\Delta\ \cos\underline{a}}$$

When either ℓ or $\frac{h}{2}$, or both are restricted, so called "compromise" designs are obtained.

(e) Find the terminal impedance Z_o where Z_o is given by

$$Z_o = 120\ \ln\frac{\lambda\sin\underline{a}}{2\pi r} - (72 + j170) = 276\ \log\frac{\lambda^m\sin\underline{a}}{d''} + 231 - j170$$

where λ^m denotes λ in meters and d'' denotes the wire diameter

in inches.

(f) Find the free-space radiation impedance for the known values of ℓ and $\underline{a}$

(g) Find the attenuation factor at the load Z_o from the formula

$$f_o^{\,2} = \epsilon^{-4\alpha\ell} = \epsilon^{-R_r/R_o} \quad \text{where} \quad \alpha = \frac{R_r}{4\ell R_o}$$

nepers per meter of loop length.

(h) Find the approximate mean square of the spatial distribution of current along the antenna from the formula

$$f_m^{\,2} = \frac{1 - f_o^{\,2}}{4\alpha\ell} = \frac{R_o}{R_r}\,(1 - f_o^{\,2})$$

(i) Determine the driving point impedance

$$Z_{in} = Z_o + j\,f_m^{\,2}\,X_r.$$

(j) Compute the db terminal loss T

$$T = -10 \log \frac{R_r f_m^{\,2}}{R_{in}} = 10 \log (1 - f_o^{\,2})^{-1}$$

(k) Write the directivity

$$g_d = \frac{1184.36\, n^2 \sin^2 \underline{a}}{R_r} \cdot \frac{\sin^4 \beta}{\beta^2}, \qquad \beta = \frac{n\pi}{4}(1 - \cos \Delta \cos \underline{a})$$

$$G_d = 10 \log g_d$$

(l) Write the power gain above ground

$$G_p = G_d - T + 6 + 20 \log \left[\sin\left(\frac{kh}{2} \sin \Delta\right) \right] \quad \text{db.}$$

(m) Finally determine the gain over a half-wave antenna in the same position

$$G_{p\frac{\lambda}{2}} = G_d - T - 2.2 \quad \text{db.}$$

III. Theoretical Considerations

The theoretical basis for the design procedure laid down above is dependent on the work of several authors extending over a score of years beginning with the work of E. Bruce[1 2]. The conditions assumed in the development of the formulae are, for the most part, highly idealized. However, it seems that these are the best approximations presently available. The following section briefly states some of the factors involved in rhombic antenna design, points out the assumptions made in the development of formulae and gives reference to the sources of this material.

(a) For $\ell < 2\lambda$ the radiation impedance and hence efficiency drops off rather rapidly. This fact is brought out in Figures 5 and 7. Hence it is good engineering practice to keep $\ell \nless 2\lambda$ at all frequencies of interest.

(b) The antenna designer must be concerned with the proper choice of the angle of radiation Δ if the antenna is to take its proper role in the transmission link. A study of ionospheric data will usually indicate a good compromise for Δ.

(c) The radiation intensity for the free-space rhombic antenna is given by

$$|K| = \frac{120\,k^2\ell^2}{\pi} \cdot \frac{W_o}{R_o} \cdot \frac{\sin^2\beta_1}{\beta_1} \cdot \frac{\sin^2\beta_2}{\beta_2} \cdot \sin^2 \underline{a}$$

$$= \frac{120}{\pi} \cdot \frac{W_o}{R_o} \cdot \frac{\sin^2\beta_1}{\sin^2\psi_{1/2}} \cdot \frac{\sin^2\beta_2}{\sin^2\psi_{2/2}} \quad \sin^2 \underline{a}$$

where $W_o = \frac{1}{2} I_o^2 R_o$ is the imput power (assumed constant). The radiation intensity for a single long wire carrying progressive

undamped waves is given by:

$$|K| = \frac{30}{\pi} \frac{W_o}{R_o} \frac{\sin^2 \beta_1}{\tan^2 \psi_{1/2}}$$

For a given rhombus, i.e. for a given ℓ/λ, $\underline{a}$ and W_o the only variable terms in the expression for $|K|$ are the terms

$$\frac{\sin^2 \beta_1}{\sin^2 \psi_{1/2}} \quad \text{and} \quad \frac{\sin^2 \beta_2}{\sin^2 \psi_{2/2}}$$

The pattern for each of these terms is <u>similar</u> to that of a long wire in free-space carrying a progressive wave, in that it consists of a series of lobes of decreasing amplitude. However, the pattern for a single wire antenna is modified by a squared cardiod factor, while no such factor applies to the rhombic antenna pattern. The main lobe of the free-space rhombus is formed where the main lobes of the β_1 and β_2 factors intersect. A secondary or 1-2 lobe of the rhombic pattern is formed where the largest lobe of one factor intersects the second largest lobe of the other factor. The definition of tertiary and higher order lobes is similar. When the rhombic antenna is placed near the Earth the resultant pattern is the product of its free-space pattern and a height factor. Under these conditions, it is important that the main lobe of the free-space antenna pattern coincide with a maximum of the height factor. This is accomplished by adjustment of the antenna height h/2. If possible the design should be "optimum", that is, the secondary and tertiary lobes should be split by zeros in the height factor while the main lobe is strengthened by a maximum in the height factor. The entire radiation pattern is intrinsically complicated and, in general, is only determined approximately. Often the positions in azimuth and elevation of the lobes and

their relative magnitudes is sufficient from an engineering viewpoint. The lobe positions can be found conveniently using the method described by Laport[10] which utilizes a graphical procedure first described by Foster[8].

(d) If the design is an optimum* "alignment" design, the value of $\underline{a}$ is determined from Figure 3. However, it is sometimes preferable to choose $\underline{a}$ to maximize the radiation intensity at $(\theta = \frac{\pi}{2} - \Delta,\ \phi = 0)$. Under the hyposthesis of a uniform traveling wave of current I_o, it has been shown[3, 8] that the radiation intensity of a horizontal rhombic antenna at a distance $h/2$ above ground is given by

$$K = 960\pi I_o^2 \left(\frac{l}{\lambda}\right)^2 \frac{\operatorname{Sin}^2\beta_1}{\beta_1} \cdot \frac{\operatorname{Sin}^2\beta_2}{\beta_2} \operatorname{Sin}^2 \underline{a} \operatorname{Sin}^2\left(\frac{kh}{2}\operatorname{Sin}\Delta\right)$$

where

$$\beta_1 = \frac{kl}{2}(1-\cos\psi_1) \qquad \operatorname{Cos}\psi_1 = \operatorname{Sin}\theta \operatorname{Cos}(\phi + \underline{a})$$

and

$$\beta_2 = \frac{kl}{2}(1-\cos\psi_2) \qquad \operatorname{Cos}\psi_2 = \operatorname{Sin}\theta \operatorname{Cos}(\phi - \underline{a})$$

with ϕ and θ being polar coordinate angles. This formula has been derived for the $\phi = 0$ line coincident with the major axis of the rhombus and the entire antenna lying in the $\theta = \frac{\pi}{2}$ $(\delta = 0)$ plane. In the $\phi = 0$ plane $\beta_1 = \beta_2 = \frac{kl}{2}(1 - \cos\Delta\cos\underline{a})$ and

$$|K|_{\phi=0} = 960\, I_o^2\, \pi \left(\frac{l}{\lambda}\right)^2 \frac{\operatorname{Sin}^4\beta}{\beta^2} \operatorname{Sin}^2 \underline{a} \operatorname{Sin}^2\left(\frac{kh}{2}\operatorname{Sin}\Delta\right)$$

In general $|K|$ is a function of the five variables I_o, $\frac{l}{\lambda}$, $\underline{a}$, θ and ϕ where W_o, the input power, is held constant.

*The "optimum" here referred to is in accordance with Laport's definition of optimum[10].

If $\frac{\ell}{\lambda}$ and $\underline{a}$ are fixed, the input impedance is fixed and I_o is a constant. In this case the positions (θ, ϕ) at which the lobe maxima occur can be very easily found. Also, the parameters W_o, θ, ϕ can be held constant and the maximum radiation intensity in a given direction as $\frac{\ell}{\lambda}$ and $\underline{a}$ are varied may be found. However, in this case, I_o will not be a constant since R_o is a function of the angle $\underline{a}$.

For maximum gain the design should be a "maximum field strength" design. Thus for W_o, Δ and ℓ chosen, the condition for

$$\frac{\partial |K|_{\phi = 0}}{\partial \underline{a}} = 0 \quad \text{is given by}$$

$$2\beta \cot\beta = \frac{\cos\Delta - \cos\underline{a}}{\sin^2\underline{a}\cos\Delta} + \frac{60}{R_o}\left[1 - \frac{\cos\Delta - \cos\underline{a}}{\cos\Delta \sin^2\underline{a}}\right], \underline{a} \gtrless \Delta;$$

or,

$$\frac{1}{\beta} - 2\cot\beta = \frac{\lambda}{\pi\ell}\,\frac{R_o - 60}{R_o} \cdot \frac{\cos\underline{a}}{\cos\Delta\sin^2\underline{a}}$$

Ignoring changes in input power, i.e. for constant input current,

$$\frac{\partial |K|_{\phi = 0}}{\partial \underline{a}} = 0 \quad \text{gives the condition}$$

$$2\beta\cot\beta = \frac{\cos\Delta - \cos\underline{a}}{\sin^2\underline{a}\cos\Delta} \quad \text{for a maximum field strength at}$$

an angle Δ.

Thus, in this case, the angle $\underline{a}$ can be chosen in the manner outlined above.

The value of the term $\frac{60}{R_o}\left[1 - \frac{\cos\Delta\cos\underline{a}}{\cos\Delta\sin^2\underline{a}}\right]$

is not likely to exceed 30% of the term $\frac{\cos\Delta - \cos \underline{a}}{\cos\Delta \, \mathrm{Sin}^2 \, \underline{a}}$.

For a fixed Δ the right hand side of the expression

$$2\beta \cot\beta = \frac{\cos\Delta - \cos \underline{a}}{\mathrm{Sin}^2 \underline{a} \, \cos\Delta} + \frac{60}{R_o} \left[1 - \frac{\cos\Delta - \cos \underline{a}}{\cos\Delta \, \mathrm{Sin}^2 \, \underline{a}} \right]$$

varies slowly with $\underline{a}$ over the range of interest.

Over this range of values of the angle $\underline{a}$ the expression $2\beta\cot\beta$ is rapidly decreasing as $\underline{a}$ increases. Thus the value of $\underline{a}$ that satisfies the equation is changed very little by a relatively large change in the value of the right hand side of the equation. In the alignment design $I_o \cdot \frac{\ell}{\lambda}$ and $\underline{a}$ are held constant and Δ is determined by $\frac{\partial |K|}{\partial \Delta} = 0$.

Foster[8] shows that this principal directional maximum occurs when the condition $\beta_1 = \beta_2 = 0.3713\,\pi$ is satisfied. Thus the main lobe and the wave angle will be aligned, when $\ell = \frac{n\lambda}{4}$, for those values of the angle $\underline{a}$ that satisfy the condition

$$\mathrm{Cos}\,\underline{a} = \frac{1 - \frac{1.4852}{n}}{\mathrm{Cos}\,\Delta}$$

Summarizing:

For an alignment design choose $\underline{a}$ in accordance with the expression

$$\mathrm{Cos}\,\underline{a} = \frac{1 - \frac{1.4852}{n}}{\mathrm{Cos}\,\Delta} \; ;$$

while for a maximum field strength design $\underline{a}$ is chosen to satisfy the condition

$$2\beta \cot\beta = \frac{\cos\Delta - \cos \underline{a}}{\mathrm{Sin}^2 \, a \cos\Delta} + \frac{60}{R_o} \left[1 - \frac{\cos\Delta - \cos \underline{a}}{\cos\Delta \, \mathrm{Sin}^2 \, \underline{a}} \right] .$$

The following conclustions have been drawn from a study of the above equations as applied to typical antennas.

1. The value of $\underline{a}$ for a given ℓ and Δ for a maximum field strength design is always greater than that for an alignment design.

2. In a maximum field strength design the principal lobe maximum occurs at an angle less than Δ. Thus the maximum field strength occurs on the upper side of the main lobe where the field strength is varying relatively rapidly with changes in θ.

3. The maximum field strength design in typical cases gives an increase in gain of about 1.5 or 1.6 db over the alignment design.

4. The maximum field strength design may find its chief field of usefulness in designing rhombic antennas to operate over wide frequency ranges. In this case the maximum field strength design at the upper frequency limit gives an increased aperture angle which tends to increase the gain of the antenna at the lower frequencies.

The design conditions for a maximum output at an angle Δ are easily derived. The radiation intensity, $|K|$, is maximized with respect to variations in ℓ when $\beta = \frac{\pi}{2}$. Under this condition the equation $\frac{\partial |K|}{\partial \underline{a}} = 0$ is satisfied when $\underline{a} = \Delta$. - exactly when the input current I_o is held constant and very nearly when the input power is held

constant. More precisely, $\underline{a}$ and Δ must satisfy the condition

$$\frac{60}{R_o - 60} = \frac{\cos \underline{a} - \cos \Delta}{\operatorname{Sin}^2 \underline{a} \cos \Delta}$$

for constant input power. The length ℓ is determined from the condition $\beta = \frac{\pi}{2}$.

The maximum output alignment design is determined similarly. In this case the directional maximum occurs when

$$\beta = 0.3713\,\pi = \frac{k\ell}{2}(1 - \cos\Delta \cos \underline{a}).$$

With this value of β, ℓ can be eliminated from the expression for $|K|$ and the condition $\frac{\partial |K|}{\partial \underline{a}} = 0$ gives the relationship

$$\frac{R_o - 60}{R_o} = \frac{\cos\Delta}{\cos \underline{a}} \; \frac{1 - \cos^2 \underline{a}}{1 - \cos\Delta \cos \underline{a}}$$

that must be satisfied when the power input is held constant. Thus summarizing the conditions for a maximum output design for a constant input power we have

(1) Maximum field strength at an angle Δ

$$\frac{60}{R_o - 60} = \frac{\cos a - \cos\Delta}{\operatorname{Sin}^2 \underline{a} \cos \Delta},$$

where R_o can be determined for $\underline{a} = \Delta$ and $\underline{a}$ redetermined from this equation. The side element length ℓ is determined from the equation $\beta = \frac{\pi}{2} = \frac{k\ell}{2}(1 - \cos\Delta \cos \underline{a})$.

(2) Alignment design (Maximum output)

$$\frac{R_o - 60}{R_o} = \frac{\cos\Delta}{\cos \underline{a}} \; \frac{1 - \cos^2 \underline{a}}{1 - \cos\Delta\cos \underline{a}}\,; \; \ell = \frac{.3713\,\lambda}{1 - \cos\Delta \cos \underline{a}}.$$

(e) At the non-generator end of the antenna it is desirable to place a load that will, as nearly as possible, eliminate reflections.

[illegible]

$$[illegible]$$

[illegible]

[illegible]

[illegible]

[illegible]

$$[illegible]$$

[illegible]

[illegible]

$$[illegible]$$

[illegible]

[illegible]

[illegible]

$$[illegible] \quad (\ldots)$$

$$[illegible]$$

[illegible]

[illegible]

[illegible]

$$[illegible] \quad (\ldots)$$

$$[illegible]$$

[illegible] $(\ldots)$

[illegible]

Looking back into the antenna from the termination it should appear as an infinite line formed by two inclined wires. From Schelkunoff's non-uniform line theory (Schelkunoff [15] [16]) a rough approximation for Z_o is given by

$$Z_o = 120 \ln \frac{\sin \underline{a}}{kr} - (72 + j170)$$

where r is the wire radius.

This can be rewritten as $276 \log \frac{\lambda^{m} \sin \underline{a}}{d''} + 231 - j170$,

where λ and d are in more convenient units as previously listed.

(f) Postulating an unattenuated traveling wave of current in the rhombic antenna the generalized circuit (Chaney[4]) for the free-space antenna has been integrated (Chaney[5]) to yield the following formula for the radiation inpedance in free-space:

$$\begin{aligned}
\frac{Z_r}{120} = \; & C + 2 \ln(2k\ell \sin^2 \underline{a}) + 2\,Ci\, 2k\ell - 2\,Ci(2k\ell \sin \underline{a}) - Ci[2k\ell(1+\cos \underline{a})] \\
& - Ci[2k\ell(1-\cos \underline{a})] \\
& + \cos(2k\ell \sin^2 \underline{a}) \Big\{ Ci[2k\ell \cos \underline{a}(1+\cos \underline{a})] + Ci[2k\ell \cos \underline{a}(1-\cos \underline{a})] \\
& + Ci[2k\ell \sin \underline{a}(1+\sin \underline{a})] + Ci[2k\ell \sin \underline{a}(1-\sin \underline{a})] - 2\,Ci[2k\ell \cos^2 \underline{a}] \\
& - 2Ci[2k\ell \sin^2 \underline{a}] \Big\} \\
& - \sin(2k\ell \sin^2 \underline{a}) \Big\{ Si[2k\ell \cos \underline{a}(1+\cos \underline{a})] - Si[2k\ell \cos \underline{a}(1-\cos \underline{a})] \\
& - Si[2k\ell \sin \underline{a}(1+\sin \underline{a})] + Si[2k\ell \sin \underline{a}(1-\sin \underline{a})] - 2\,Si[2k\ell \cos^2 \underline{a}] \\
& + 2\,Si[2k\ell \sin^2 \underline{a}] \Big\} \; . \\
& + j\Big[Si[2k\ell(1+\cos \underline{a})] - Si[2k\ell(1-\cos \underline{a})] + 2\,Si[2k\ell \sin \underline{a}] - 2\,Si\, 2k\ell \\
& - \cos(2k\ell \sin^2 \underline{a}) \Big\{ Si[2k\ell \cos \underline{a}(1+\cos \underline{a})] + Si[2k\ell \cos \underline{a}(1-\cos \underline{a})]
\end{aligned}$$

$$+ \mathrm{Si}[2k\ell \sin \underline{a}(1+\sin \underline{a})] + \mathrm{Si}[2k\ell \sin \underline{a}(1-\sin \underline{a})] - 2\,\mathrm{Si}[2k\ell \cos^2 \underline{a}]$$

$$- 2\,\mathrm{Si}[2k\ell \sin^2 \underline{a}]\Big\}$$

$$- \mathrm{Sin}(2k\ell \sin^2 \underline{a})\Big\{\mathrm{Ci}[2k\ell \cos \underline{a}(1+\cos \underline{a})] - \mathrm{Ci}[2k\ell \cos \underline{a}(1-\cos \underline{a})]$$

$$- \mathrm{Ci}[2k\ell \sin \underline{a}(1+\sin \underline{a})] + \mathrm{Ci}[2k\ell \sin \underline{a}(1-\sin \underline{a})] - 2\,\mathrm{Ci}[2k\ell \cos^2 \underline{a}]$$

$$+ 2\,\mathrm{Ci}[2k\ell \sin^2 \underline{a}]\Big\}\Big]$$

The above formula gives the radiation impedance of the free-space rhombic antenna. However, if $h/2$ is greater than $0.4\,\lambda$ this value and the actual value should agree fairly well (Lewin[12]).

(g) In the derivation of an expression for the attenuation constant uniform transmission line theory may be applied to yield the result $\alpha = \frac{1}{2}\frac{W_L}{W_T}$ (Ramo and Whinnery[14]), where W_L is the time average power loss per unit length and W_T is the time average power transmitted on an infinite line (i.e. by an unattenuated traveling wave). As is customary when transmission losses are small, it will be assumed that no attenuation of current occurs in determining W_L and W_T for obtaining a first order approximation for α.

The driving point impedance of the rhombic element of loop length 2ℓ is given by generalized circuit theory (Chaney[4]) as

$$Zin = 2\ell Z_i f_m^2 + j\frac{30}{k}\oint_1\oint_2 \mathrm{Re}[f(P_1)^* f(P_2)]\,\bar{\bar{\nabla}}_1[e(r_{12})\,d\bar{r}_2]\cdot d\bar{r}_1 + f_0^2 Z_0 .$$

For a rhombus using $f_m^2\, g(P_1P_2) = \mathrm{Re}[f(P_1)^* f(P_2)]$,

$$Zin = 2\ell Z_i f_m^2 + j\frac{30}{k} f_m^2 \oint_1 \oint_2 g(P_1P_2) \bar{\bar{\nabla}} [e(r_{12}) d\bar{r}_2] \cdot d\bar{r}_1$$

$$+ f_o^2 Z_o = 2\ell Z_i f_m^2 + f_m^2 Z_r + f_o^2 Z_o .$$

However, as pointed out by Chaney[6], if the spatial rate at which the reactive power due to the localized non-retarded fields is reentering the wire is taken into account, additional terms must be included and the following more correct expression is deduced:

$$Z_{in} = Z_o + j f_m^2 (2\ell x_i + x_r).$$

Neglecting the internal reactance $Z_{in} = Z_o + j f_m^2 X_r$.

Following a suggestion of Lewin[12], assume the radiation impedance to be distributed as a series internal impedance. Thus, before attenuation,

$$Zin = 2\ell f_m^2 (Z_i + \frac{Z_r}{2\ell}) + f_o^2 Z_o + j(1 - f_o^2) X_o - 2\ell R_i f_m^2 ;$$

where the last two terms are the corrections due to the previously neglected localized non-retarded fields mentioned above. The series impedance per unit loop length is $Z_i + \frac{Z_r}{2\ell}$. The time average loss per unit length (before attenuation) is $\frac{1}{2} I_o^2 (R_i + \frac{R_r}{2\ell})$. The time average power transmitted under an assumed perfectly matched condition is $W_T = \frac{1}{2} I_o^2 R_o$.

Hence $\alpha = \frac{1}{2} \frac{W_L}{W_T} = \frac{R_r + 2\ell R_i}{4\ell R_o} \approx \frac{R_r}{4\ell R_o}$ if $R_r >> 2\ell R_i$.

Since the terminal impedance, Z_o , is usually considered as a lumped parameter , $f_o^2 = \epsilon^{-4\alpha\ell} = \epsilon^{-\frac{R_r}{R_o}}$.

The attenuation factor $\alpha = \frac{R_r}{4\ell R_o}$ accounts for the attenuation due to the radiation of energy along the wire. However, there is an additional attenuation factor. If we consider the rhombic antenna

as being made up of two sections of non-uniform transmission line matched at their junction, then each section of the line has an approximate attenuation factor given by $m\alpha$ which is positive in one case and negative in the other. It is interesting to note that, if α is small, the value of f_o^2, i. e. the current at the termination, is unaffected by the attenuation factor of the non-uniform sections. The value of current everywhere else in the antenna is, however, affected.

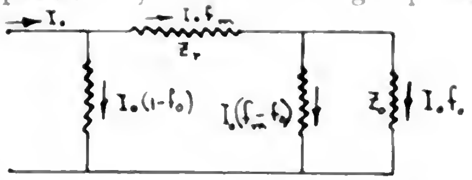
(h) Assuming the current to be given by $I = I_o\,\epsilon^{-(\alpha + j\beta)x}$,

the mean square current modulus is given by

$$f_m^2 = \frac{1}{2}\int_0^{2\ell} |\epsilon^{-\alpha x}|^2\, dx = \frac{1}{4\alpha\ell}(1-\epsilon^{-4\alpha\ell}) = \frac{R_o}{R_r}(1-f_o^2).$$

(i) Again, from generalized circuit theory, neglecting Z_1 the expression for the input impedance becomes $Z_{in} = Z_o + j\frac{f_m^2 x_r}{\ }$.

(j) In view of the approximations made already, the rhombic element may be replaced by the following equivalent circuit



The total useful power -- i. e. the total power available for radiation, is $\frac{1}{2}I_o^2 f_m^2 R_r + \frac{1}{2}I_o^2 f_o^2 R_o$.

Power radiated is $\frac{1}{2}I_o^2 f_m^2 R_r$

Terminal loss is $10\log\frac{f_m^2 R_r + f_o^2 R_o}{f_m^2 R_r} = 10\log\frac{R_{in}}{f_m^2 R_r} \rightarrow 10\log(1-f_o^2)^{-1}$.

(k) The directivity is given by $g_d = \dfrac{4\pi K_{max}}{\frac{1}{2} I_o^2 R_r}$

where K is the radiation intensity and K_{max} and R_r are referred to the same current. If $\underline{a}$ has been selected as prescribed above K_{max} lies in the $\phi = 0$ plane, **and, under** the assumption of uniform unattenuated current in the rhombus, is given by

$$|K_{max}|_{\phi=o} = 240\pi I_o^2 \left(\frac{l}{\lambda}\right)^2 \frac{\sin^4\beta}{\beta^2}. \sin^2 \underline{a}$$

where the symbols are listed at the outset. Now the postulation of uniform unattenuated current implies no radiation. Hence, it would seem more desirable to use a formula for K_{max} based on an assumed attenuated current wave. Hoffman[9] has obtained a general expression for the field distribution from a rhombic antenna carrying damped progressive waves. A comparison of his formula with the one given above shows that for small values of the attenuation factor, α, the main difference is that the field does not fall to zero at the minima in the case of attenuated current. However, the factor $e^{-2\alpha l}$ appears in the expression for $|K_{max}|_{\phi = 0}$. A closer approximation to the actual current producing K would be $I_o f_m$ and not I_o. Indeed, Christiansen[7] indicates that when the average current is used in the approximate formula above, the results agree closely with those given by the more exact methods of Hoffman. The fact that the actual current producing K is approximated more closely by $I_o f_m$ than

by I_o is taken into account below, when, in computing the gain, the terminal loss is subtracted.

Thus

$$K_{max} = 240\pi \, . \, I_o^2 \left(\frac{\ell}{\lambda}\right)^2 \frac{\sin^4\beta}{\beta^2} . \sin^2 \underline{a}$$

It is further assumed that $\ell = \frac{n\lambda}{4}$.

$$\text{Thus } g_d = \frac{240 \, . \, 8\pi \, . \, \pi \, . \, n^2 \sin^2 \underline{a}}{16 \, R_r} \quad \frac{\sin^4\beta}{\beta^2}$$

$$= \frac{1184.36 \, n^2 \sin^2 \underline{a}}{R_r} \quad \frac{\sin^4\beta}{\beta^2}, \quad \beta = \frac{n\pi}{4}(1 - \cos\Delta \cos \underline{a}),$$

and

$$G_d = 10 \log g_d .$$

(l) Since all the radiated power is radiated into a hemisphere above ground, the power gain above ground is given by

$$G_p = G_d - T + 6 + 20 \log \left[\sin\left(\frac{kh}{2} \sin \Delta\right)\right] \text{ db} .$$

(m) The gain of a half-wave dipole in free-space is 2.2 db. Hence the gain of a rhombic antenna over a half-wave dipole in the same position is

$$G_{p\frac{\lambda}{2}} = G_d - T - 2.2 \quad \text{db.}$$

In concluding this section it might be worthwhile to try to clarify the connection between directivity and terminal loss. Assuming, as some authors do, that the power lost in the terminating resistor must be attributed to the antenna, it is possible to formulate a "modified" directivity expression as follows:

$$\text{Total Power delivered} = \frac{1}{2} I_o^2 f_m^2 R_r + \frac{1}{2} I_o^2 f_o^2 R_o$$

$$= \frac{1}{2} I_o^2 \left\{ R_o(1 - f_o^2) + f_o^2 R_o \right\}$$

$$= \frac{1}{2} I_o^2 R_{in}$$

The radiation intensity under conditions of attenuated current is approximately $f_m^2 K_{max}$ where K_{max} is determined for an assumed constant current amplitude.

$$\text{"Modified" Directivity} = \frac{4\pi f_m^2 K_{max}}{\frac{1}{2} I_o^2 R_{in}}$$

$$= \frac{4\pi K_{max}}{\frac{1}{2} I_o^2 R_r} \cdot \frac{f_m^2 R_r}{R_{in}}$$

$$= \frac{\text{Directivity}}{\text{Terminal Loss}} .$$

Hence the gain of the antenna $= 10 \log \dfrac{4\pi K_{max}}{\frac{1}{2} I_o^2 R_r} - T.$

It should be pointed out that this equation does not imply that the power radiated is $\frac{1}{2} I_o^2 R_r$, rather it has been derived for an assumed radiated power $= \frac{1}{2} I_o^2 f_m^2 R_r$.

Strictly speaking the radiation intensity under conditions of attenuated current is more closely approximated, in the $\phi = 0$ plane, by the expression $\epsilon^{-2\alpha\ell}|K|$ where K is given above. This approximate expression is not satisfactory in the neighborhood of minima. However, the "modified directivity" is in close agreement with that which would be obtained by using $\epsilon^{-2\alpha\ell} K_{max}$ for the radiation intensity.

For example,

$$f_m^{\,2} = \frac{R_o}{R_r}\left(1 - \epsilon^{-\frac{R_r}{R_o}}\right) = 2\frac{R_o}{R_r}\,\epsilon^{-\frac{R_r}{2R_o}}\;\frac{\epsilon^{\frac{R_r}{2R_o}} - \epsilon^{-\frac{R_r}{2R_o}}}{2}$$

$$= \epsilon^{-\frac{R_r}{2R_o}}\;\frac{\sinh \frac{R_r}{2R_o}}{\frac{R_r}{2R_o}}$$

Then, since $\frac{R_r}{2R_o} = 2\alpha\ell$ and since in practice R_r seldom appreciably exceeds R_o such that $\frac{\sinh \frac{R_r}{2R_o}}{\frac{R_r}{2R_o}} < \frac{\sinh 0.5}{0.5} = 1.04$

it follows that $f_m^{\,2} \approx \epsilon^{-2\alpha\ell} = f_o$. Thus, errors in gain calculations from this cause should not exceed 0.18 db.

A similar demonstration shows that the square of the mean current is approximately the same as the mean square current. Since the mean current is

$$\frac{1}{2\alpha\ell}\left(1 - \epsilon^{-2\alpha\ell}\right) = \epsilon^{-\alpha\ell}\;\frac{\sinh \frac{R_r}{4R_o}}{\frac{R_r}{4R_o}}$$

its square likewise is very close to $\epsilon^{-2\alpha\ell} = f_o$.

IV. Design Aids

The theoretical design of a horizontal rhombus may be aided considerably if the proper information is available in a tabulated or graphical form. Within this section a few graphical design aids are presented. Reference is here made to Laport's[10] article for tabulated data on field patterns of rhombic antennas.

Figure 2 is a plot of height factor nulls and maximums versus electrical height in degrees. Figure 3 shows a plot of the optimum

parameters for horizontal rhombic antennas for maximum gain and minimum side lobe amplitudes. Where possible Figure 3 should be used as much time can be saved in this manner. These parameters are optimum for one frequency only. The data was compiled by Laport[10] from stereographic charts. An example of the use of this figure will be given in the next section.

Figure 4 is a plot of $\underline{a}$ vs Δ, for various values of n, to satisfy the condition for maximum radiation intensity at Δ for constant input current. The condition referred to is

$$2\beta \cot \beta = \frac{\cos \Delta - \cos \underline{\underline{a}}}{\sin^2 \underline{a} \cos \Delta}$$

It must be stressed however that Figure 4 gives approximate values only. These are usually accurate enough. If, however, greater accuracy is desired Figure 4 may be used to give the proper "neighborhood" of the correct value of $\underline{a}$. With the approximate value the correct value may be quickly determined by plotting both sides of the equation in the neighborhood of the approximate value.

Figure 4a gives the value of $\underline{a}$ for various n for alignment of the main lobe with the wave angle Δ. Figure 4a shows how the elevation of the main lobe of a rhombus varies with leg length and $\underline{a}$. The main beam splits when $\underline{a}$ exceeds the angle of the first maximum of the

$$\frac{\operatorname{Sin}^2 \beta}{\beta} \quad \text{pattern.}$$

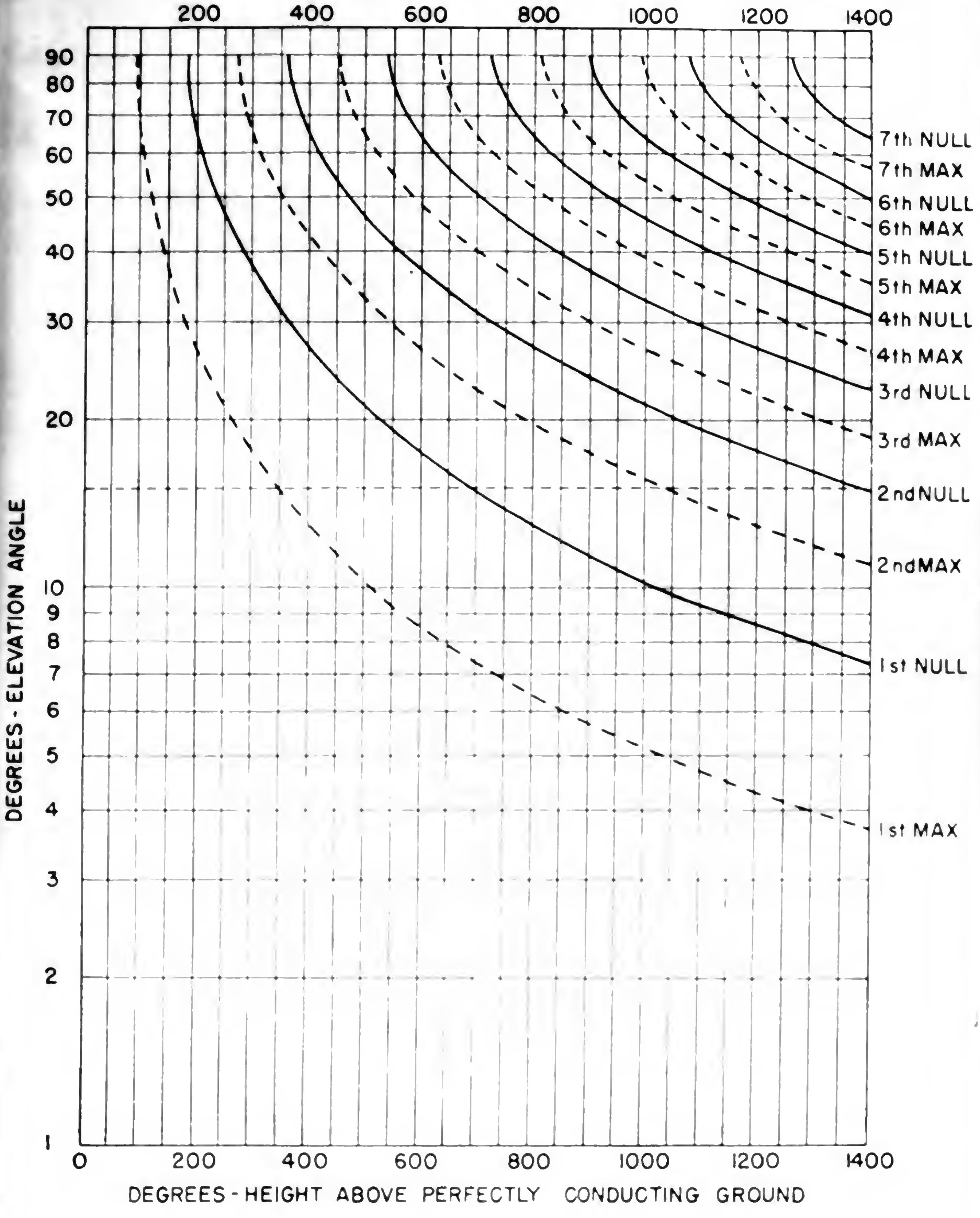


Figure 2.

By permission from "Radio Antenna Engineering", by E. A. Laport. Copyright, 1952. McGraw-Hill Book Company, Inc.

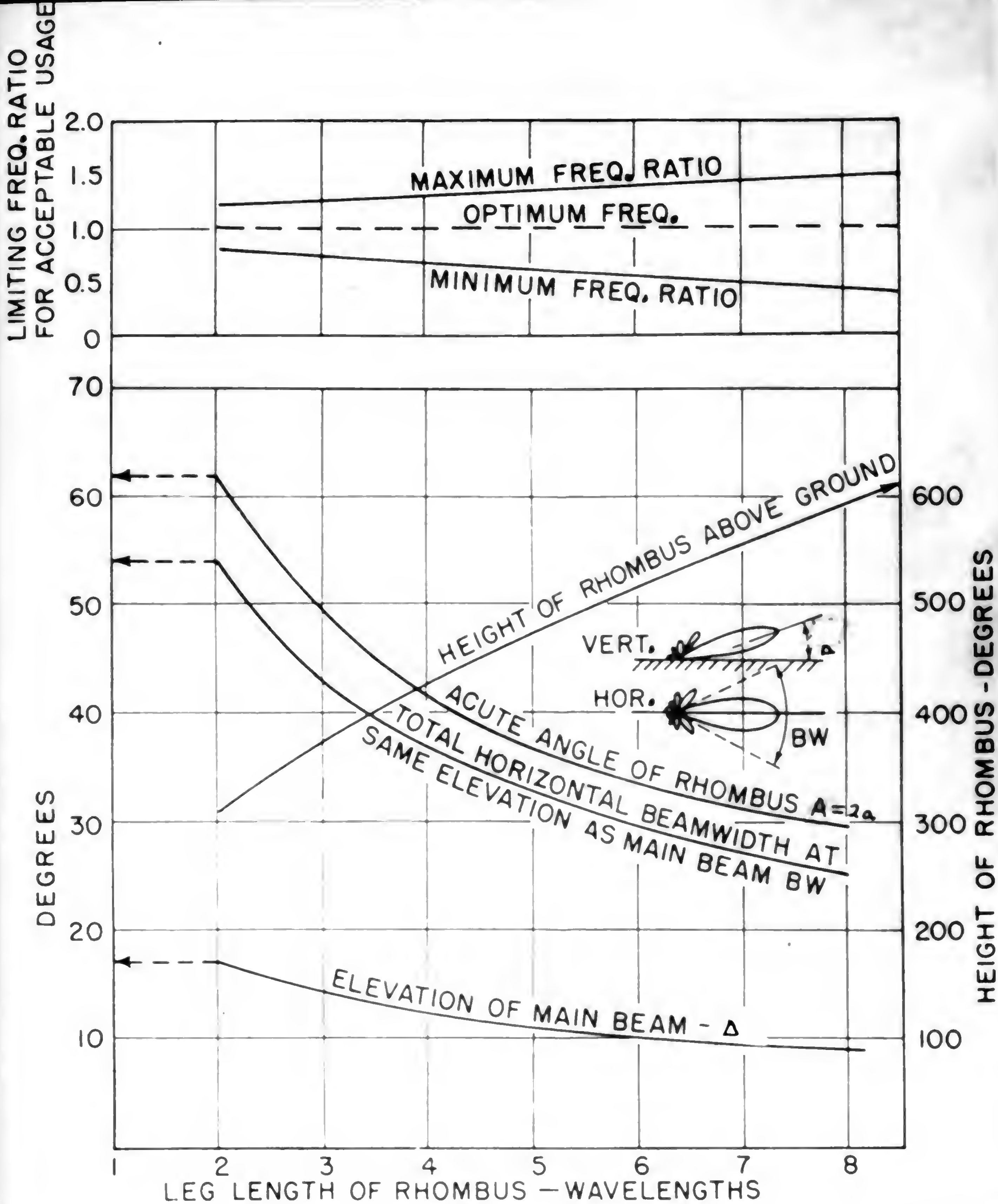

LIMITING FREQ. RATIO FOR ACCEPTABLE USAGE
2.0
1.5
1.0
0.5
0
MAXIMUM FREQ. RATIO
OPTIMUM FREQ.
MINIMUM FREQ. RATIO
70
60
50
40
30
20
10
DEGREES
600
500
400
300
200
100
HEIGHT OF RHOMBUS - DEGREES
HEIGHT OF RHOMBUS ABOVE GROUND
VERT.
HOR.
BW
ACUTE ANGLE OF RHOMBUS A=2a
TOTAL HORIZONTAL BEAMWIDTH AT SAME ELEVATION AS MAIN BEAM BW
ELEVATION OF MAIN BEAM - Δ
1
2
3
4
5
6
7
8
LEG LENGTH OF RHOMBUS — WAVELENGTHS

Figure 3.

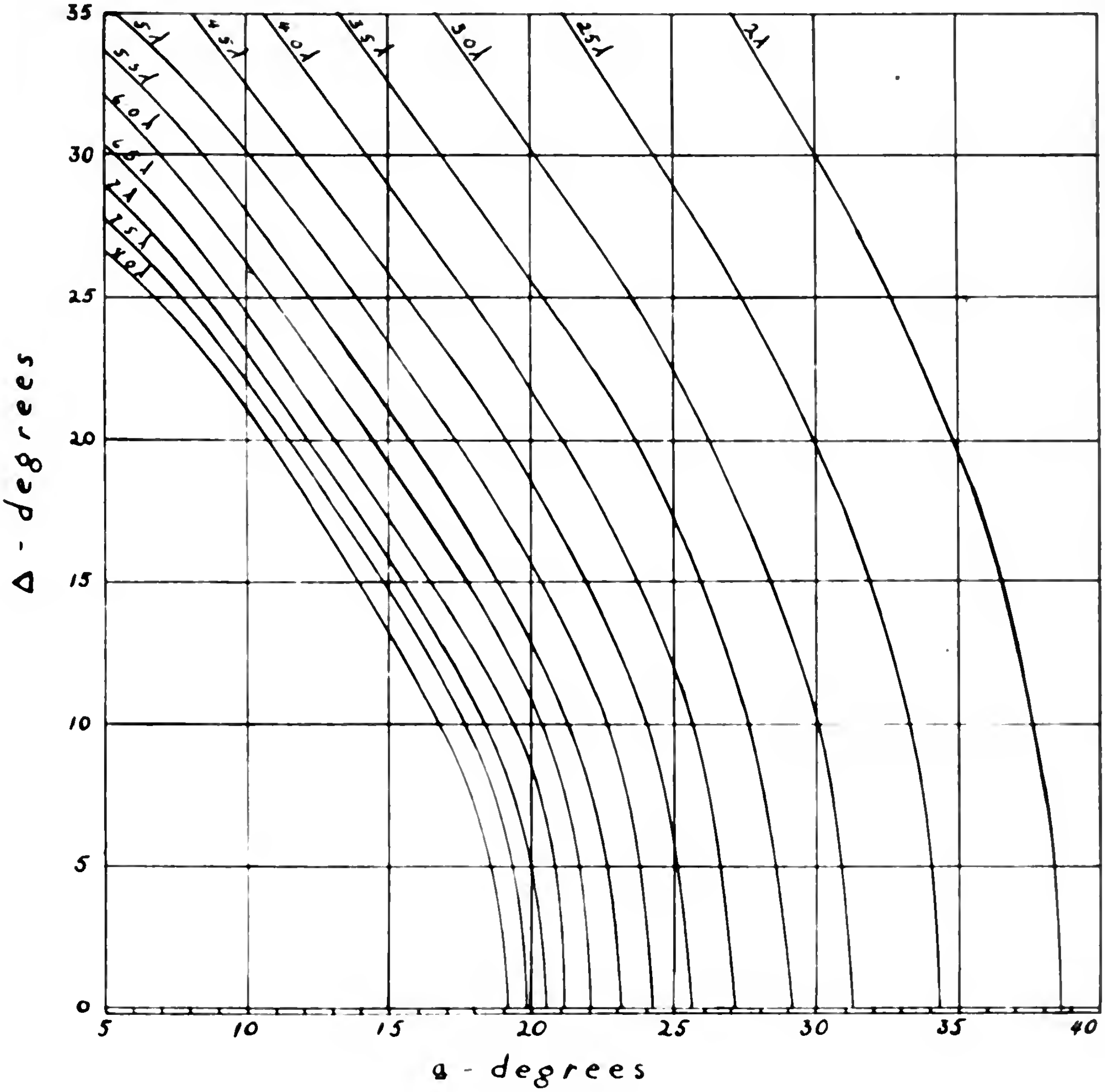


Figure 4

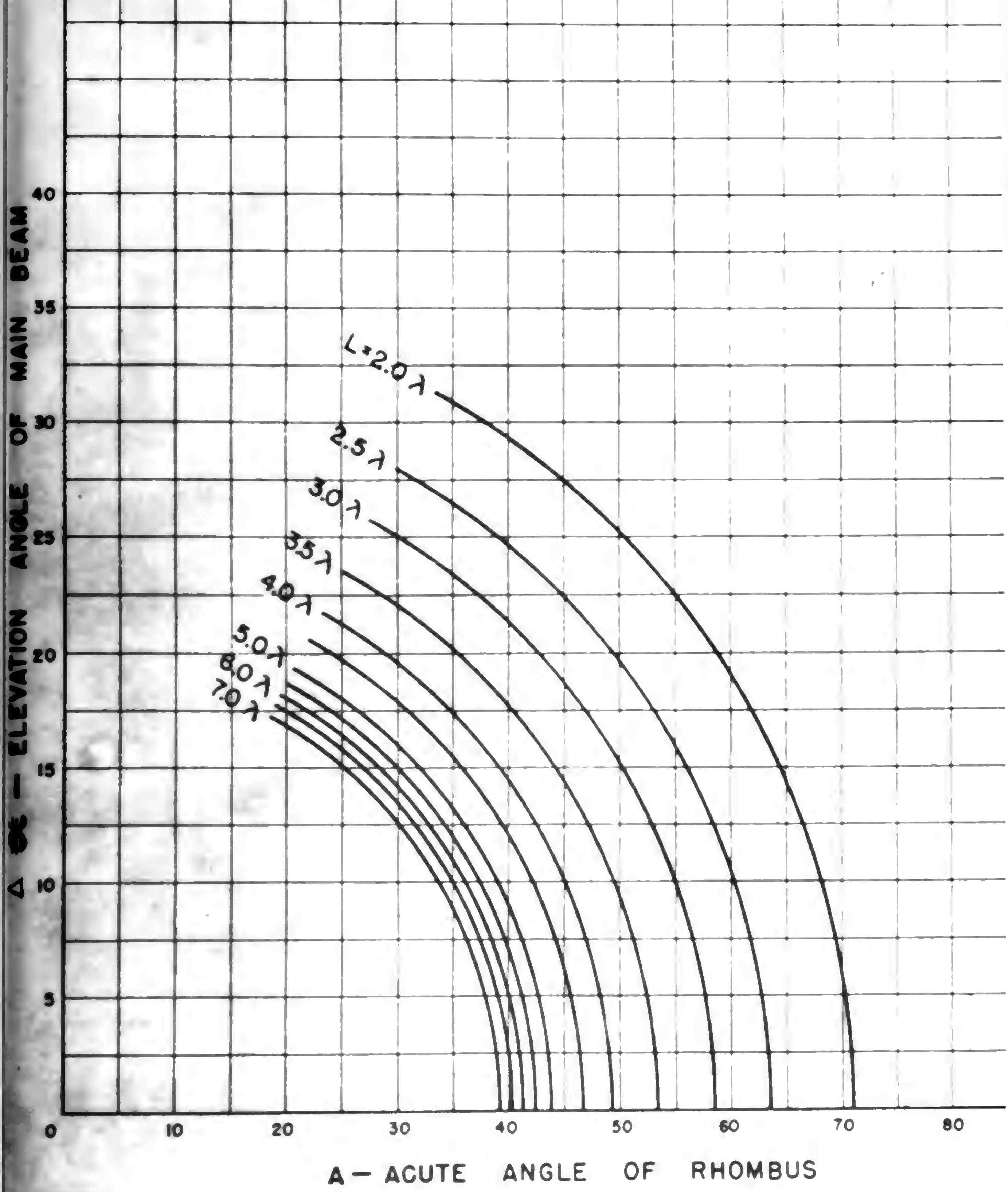


Figure 4a -Main-beam elevation with respect to the plane of a rhombus in free space.

By permission from "Design Data for Horizontal Rhombic Antennas", by E. A. Laport. Copyright, 1952, RCA Review.

Table I shows the critical values of $\underline{a}$ at which disintegration of the main lobe occurs. The angle $\underline{a}$ should never quite equal the values given in Table I if a single main lobe in the $\phi = 0$ plane is desired.

TABLE I

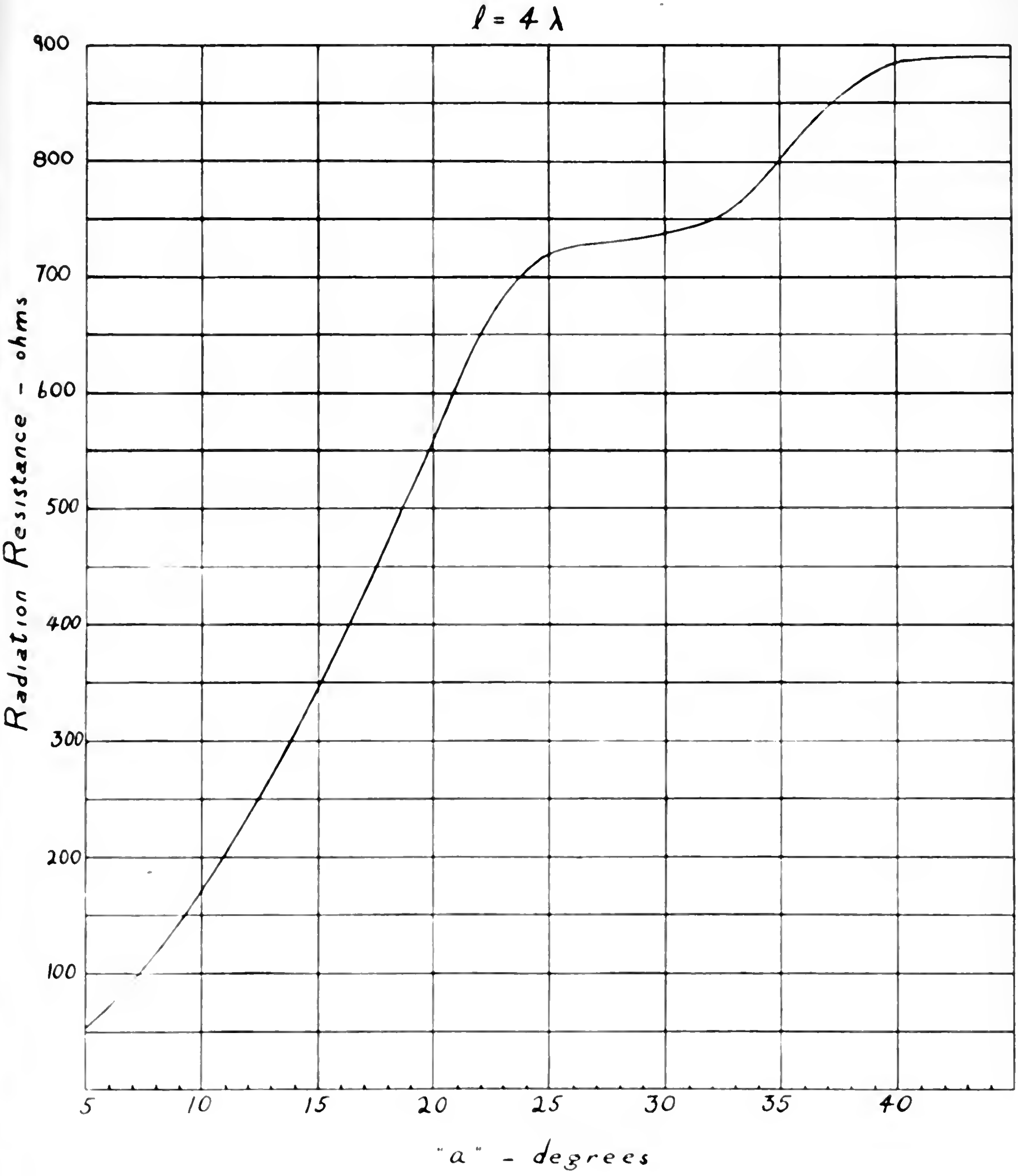
$\frac{\ell}{\lambda}$	$\underline{a}$ crit.
2	38 degrees
3	~~30~~ 31 "
4	27 "
5	24 "
6	22 "
7	20.5 "
8	19.3 "

Table II gives the relative amplitudes of the lobes in the field strength pattern for the free-space rhombic antenna with reference to the main lobe. These amplitudes are true only for the fully formed main lobe pattern.

TABLE II

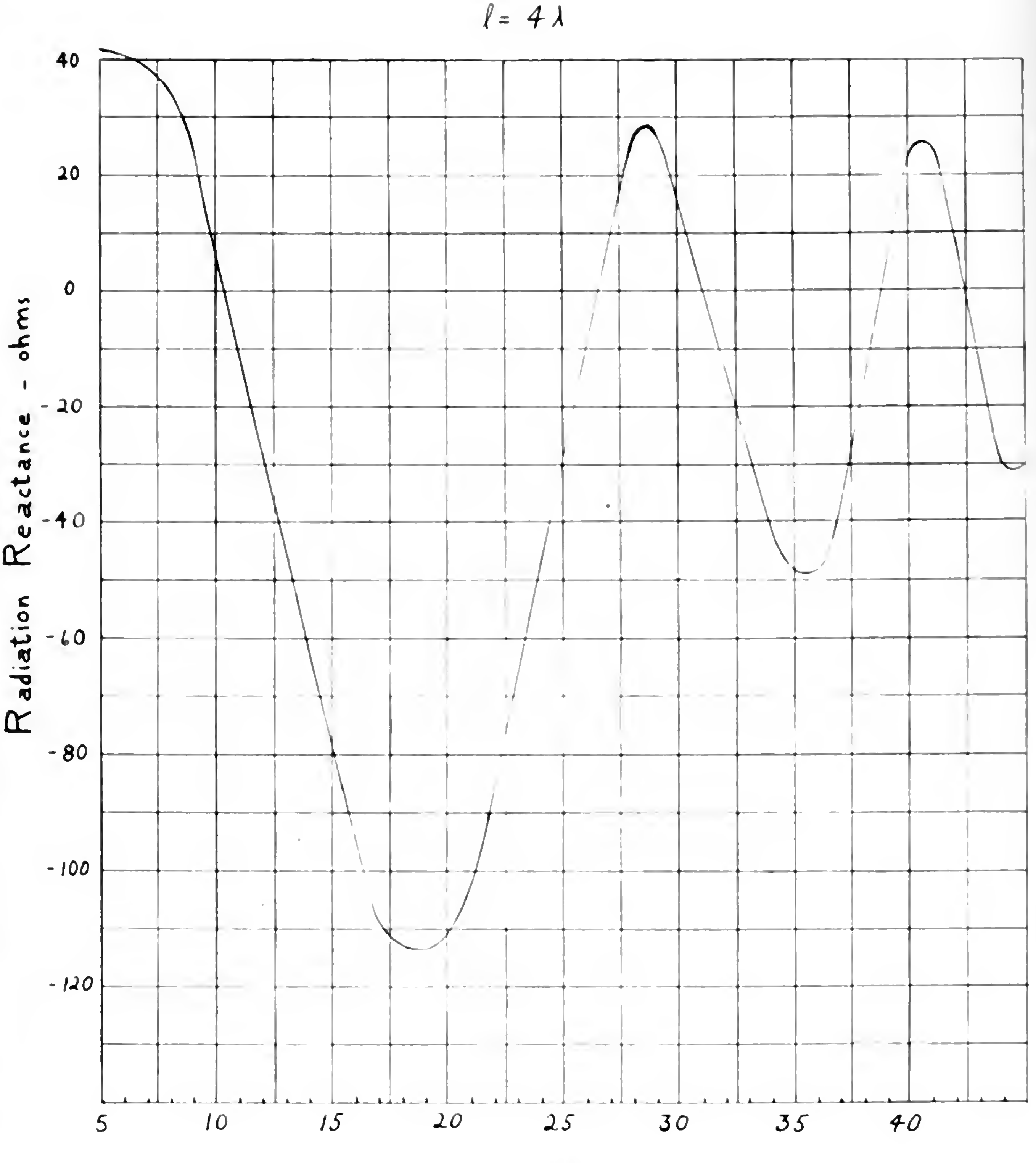
For Second Side	Order of Maximums For First Side					
	1	2	3	4	5	6
1	1.000					
2	0.544	0.26				
3	0.420	0.038	0.003			
4	0.354	0.007	0.00054	0.0001		
5	-	0.002	0.000156	0.000028	10^{-6}	
6	-	-				

Figures 5, 6, 7 and 8 gives the variations of Z_r with $\underline{a}$ and ℓ. Figure 9 is a plot of R_r vs $\underline{a}$ for constant ℓ and Δ. The "optimum" parameters are also plotted.



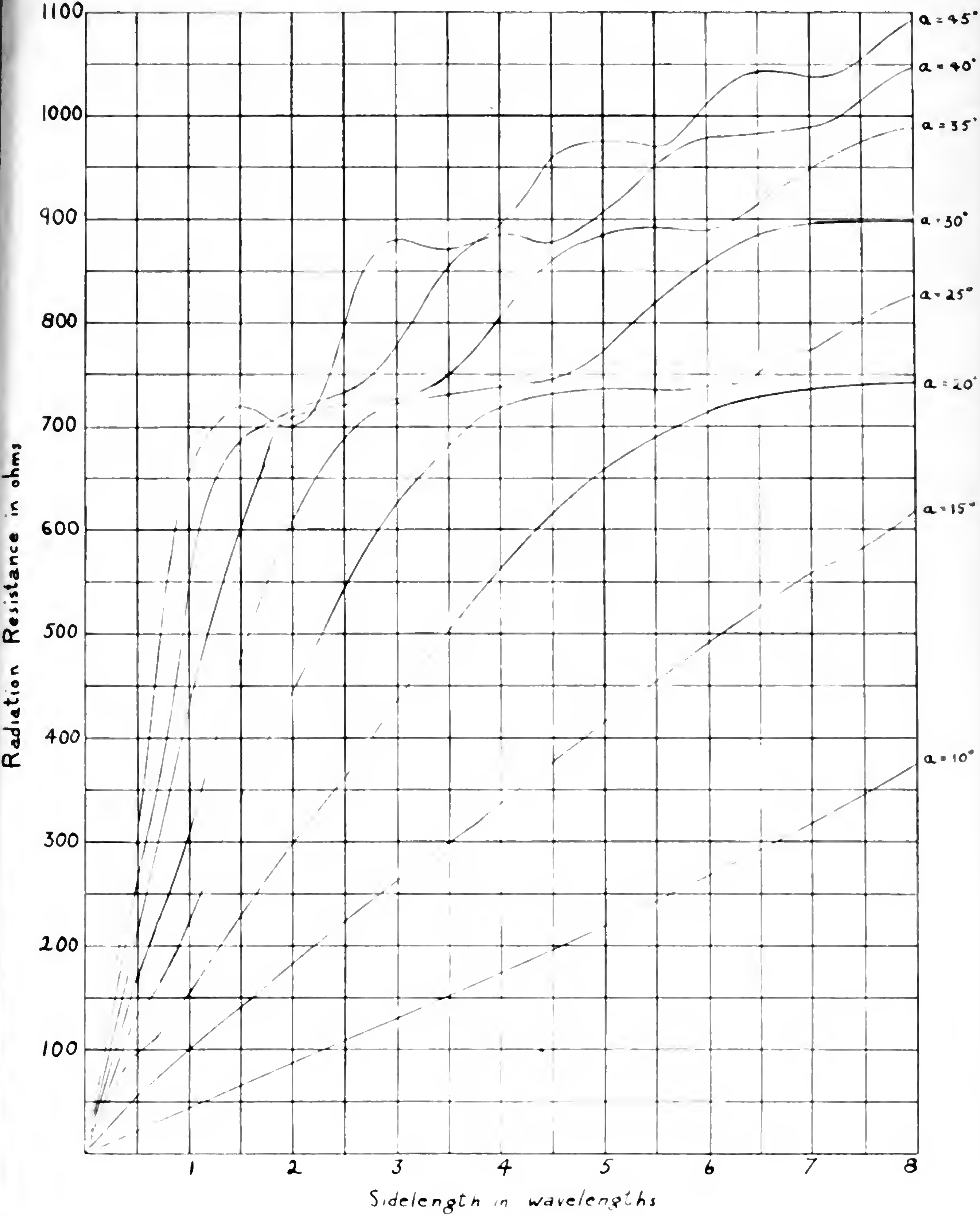
$l = 4\lambda$
900
800
700
600
500
400
300
200
100
Radiation Resistance - ohms
5
10
15
20
25
30
35
40
"a" - degrees

Figure - 5



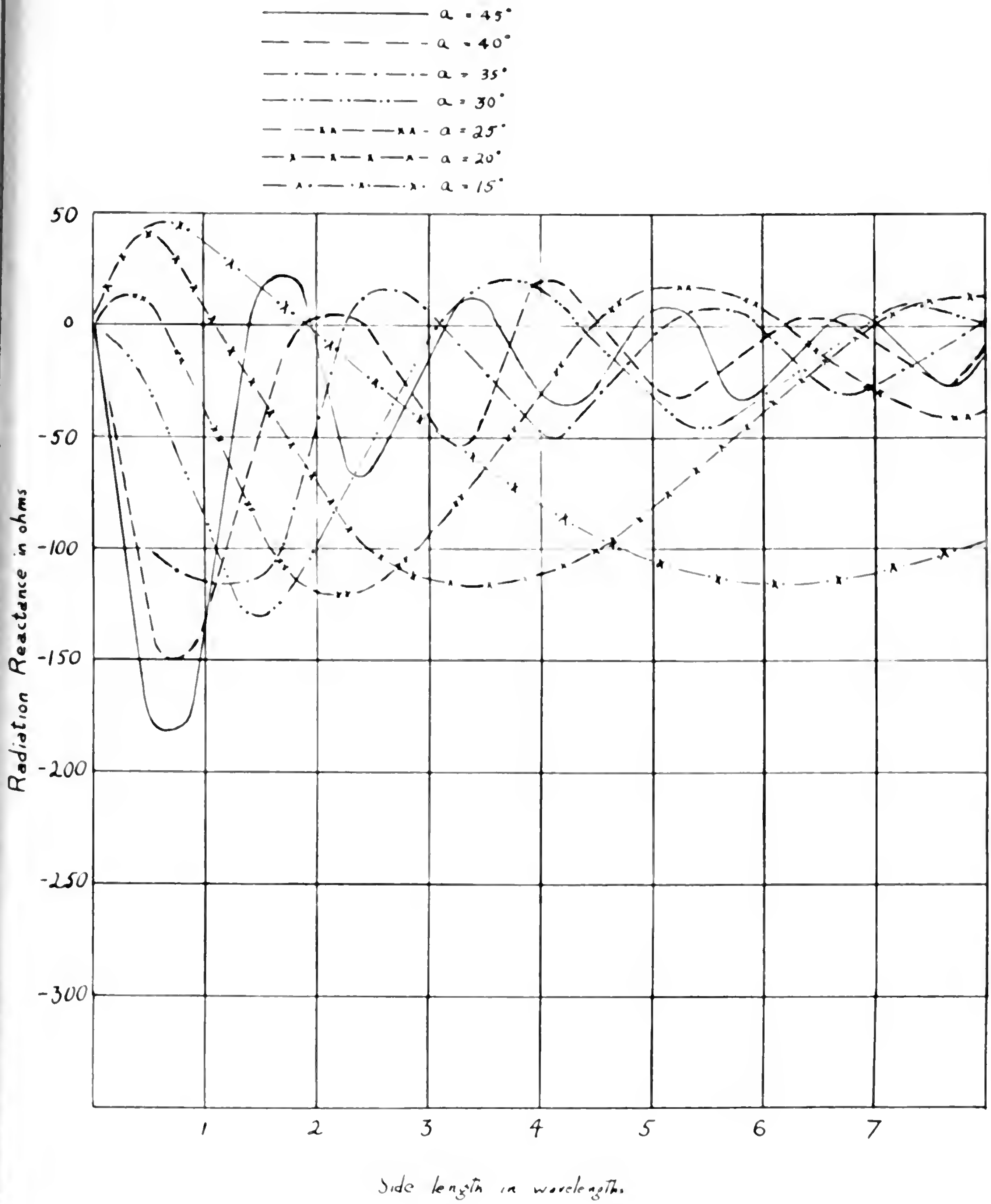
$l = 4\lambda$
40
20
0
-20
-40
-60
-80
-100
-120
Radiation Reactance - ohms
5
10
15
20
25
30
35
40
a - degrees

Figure - 6



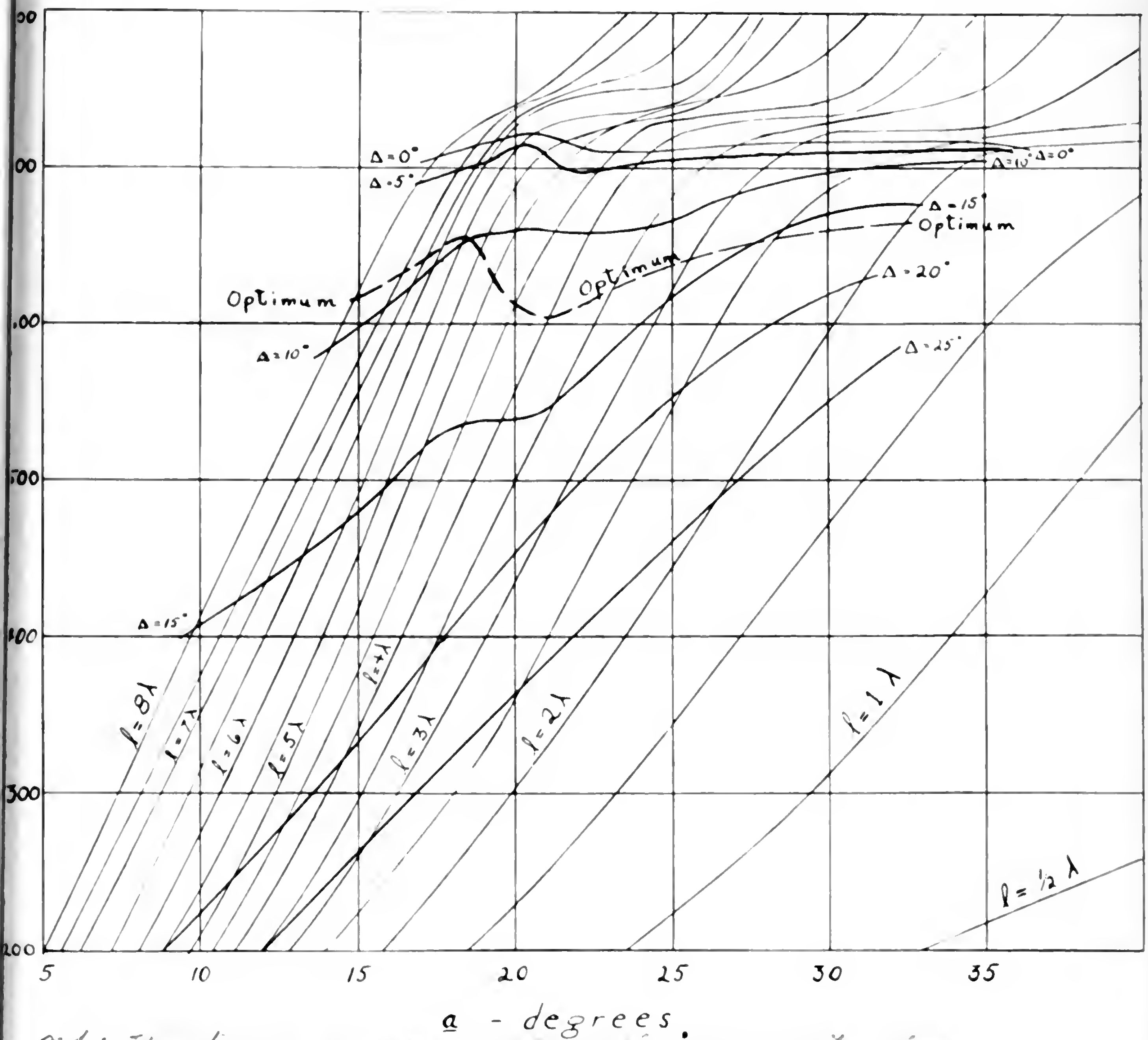
1100
1000
900
800
700
600
500
400
300
200
100
1
2
3
4
5
6
7
8
a = 45°
a = 40°
a = 35°
a = 30°
a = 25°
a = 20°
a = 15°
a = 10°
Radiation Resistance in ohms
Sidelength in wavelengths

Figure - 7



a = 45°
a = 40°
a = 35°
a = 30°
a = 25°
a = 20°
a = 15°
50
0
-50
-100
-150
-200
-250
-300
1
2
3
4
5
6
7
Radiation Reactance in ohms
Side length in wavelengths

Figure -8



note: This figure is not sufficiently accurate for computation for a < 15° r.c.

Figure - 9

V. Examples

Example 1. - - Optimum design - constant height

Assume that it is desired to have a rhombic antenna operating at f_o with its main beam at $\Delta = 12.5$ degrees. Laport[10] uses this as an example and obtains $\ell = 4\lambda$; $\underline{a} = 21$ degrees; $h/2 = 423$ degrees $= 1.18\lambda$. Total beam width between first zeros = 37 degrees. The height factor maximums from Figure 2 occur at 12.5 and 39.5 degrees while the zeros occur at 0, 25 and 57 degrees. The relative lobe amplitudes are calculated to be

1-1	1.000
1-2	0.
1-3	0.
2-3	0.23
2-4	0.00049
3-3	0.0014

Other lobes are smaller. It is assumed no lobe is less than 0.02 of the main lobe.

Let us assume a #10 wire is used i.e. $d = .102$ inches. Furthermore assume $f_o = 30$ mcs , $\lambda_o = 10$ meters.

$$Z_o = 276 \log \frac{\lambda^m \sin \underline{a}}{d} + 231 - j170 = 658 - j\,170\ \Omega.$$

$$Z_R = 595 - j\,94\ \Omega \text{ Since } h/2 > 0.4\lambda.$$

$$f_o^2 = \epsilon^{-\frac{R_r}{R_o}} = \epsilon^{-\frac{595}{658}} = .405$$

$$f_m^2 = \frac{R_o}{R_r}(1 - f_o^2) = .658$$

$$Z_{in} = R_o + j f_m^2 X_r = 658 - j\,.232\ \Omega.$$

Terminal loss $T = -10 \log (1-f_o^2) = 2.25$ db

$\ell = 4\lambda$, $n = 16$, $n^2 = 256$.

$\operatorname{Sin} \underline{a} = \operatorname{Sin} 21^\circ = .359$, $\operatorname{Sin}^2 \underline{a} = .113$.

$\operatorname{Cos} \Delta = .976$, $\operatorname{Cos} \underline{a} = .933$.

$\beta = \frac{n\pi}{4} (1-\cos \Delta \cos \underline{a}) = 4\pi(1-.912) = 1.11$ radians $= 63.3^\circ$.

$\operatorname{Sin}\beta = .893$, $\operatorname{Sin}^4\beta = .632$, $\beta^2 = 1.22$.

$g_d = 29.7$, $G_d = 10 \log g_d = 14.7$ db

$G_p = 18.4$ db

$G_{p\frac{\lambda}{2}} = 10.2$ db

Following Laport's example we now turn to the determination of the high frequency limit of this antenna.

Figure 4a shows that, in order not to split the main beam for $\underline{a} = 21^\circ$ the greatest leg length is $\ell = 5.5\lambda$. Under these conditions $\Delta = 3$ degrees. At this wavelength the electrical height is 581 degrees. The height factor maximums occur when $581 \sin\delta = 90$ and 270 degrees. The height factor nulls occur when $581 \operatorname{Sin}\delta = 0$, 180 and 360 degrees. Thus the height factor maximums are at $\delta = 8.5$ degrees and 27.7 degrees and the nulls are at $\delta = 0$, 18.0 and 38.3 degrees. The value of the height factor $f(\delta) = \operatorname{Sin}(581 \operatorname{Sin}\delta)$ at 3 degrees is 0.512. The maximum of the main lobe will occur where the product of the height factor and free-space pattern are a maximum. A rough estimation is $\Delta = 4$ degrees and a relative amplitude of 0.74. From tables in Laport's article the 1-2 lobe

occurs at $\Delta = 21$ degrees. The height factor at 21 degrees has the value 0.515. Relative to a fully formed main beam the 1-2 lobe amplitude is 0.544 (See Table II). Hence the **1-2** lobe amplitude is 0.515 x 0.544 = 0.28 referred to a fully formed main lobe. After normalizing the main lobe the 1-2 lobe amplitude is $\frac{0.28}{(.74)^2} = 0.5$ i.e. the **1**-2 lobe is down 6 db. From an interference standpoint this is a large lobe. The 1-3 lobe is a little larger than the 1-2 lobe. At this limiting frequency:

$\lambda = \frac{4}{5.5} \times 10 = 7.27$ m

$Z_o = 620 - j170$ ohms

$Z_r = 695 - j40$ ohms

$f_o^2 = .326$

$f_m^2 = .601$

$Z_{in} = 620 - j\,194$

$T = 1.72$ db

$n = 22$, $n^2 = 484$

$\mathrm{Sin}^2 \underline{a} = 0.113$; $\mathrm{Cos}\,\Delta = 0.997$; $\mathrm{Cos}\,\underline{a} = 0.833$

$\beta = 81.8$ degrees $= 1.43$ radians

$\beta^2 = 2.04$

$\mathrm{Sin}\beta = .988$, $\mathrm{Sin}^4 \beta = .95$

$g_d = 43.4$

$G_d = 16.5$ db

$G_p = 20.8$ db

$G_{p\frac{\lambda}{2}} = 12.6$ db

This example shows that, although at the high frequency the antenna is more efficient, from an interference point of view it may not be desirable to operate this antenna over a $\frac{5.5}{4}$ frequency range.

<u>Example 2 -- Non-optimum design -- fixed height</u>

Assume that from ionospheric studies it is determined that $\Delta = 4$ degrees will permit communication over a given path a certain fraction of the time. Assume further that the wave angle must not exceed 12 degrees or otherwise the signal will penetrate the ionosphere and be lost. The wavelength is 10 meters.

Examination of Figure 3 shows that an optimum design is not possible.

Following the design procedure laid down above we have:

$$\frac{h}{2} = \frac{\lambda}{4 \text{ Sin } \Delta} = \frac{10}{4 \times 0.07} = 35.7\text{m} = 1287 \text{ degrees}$$

From Figure 4 Case 1: $\underline{a} = 35.5$ degrees for $\ell = 2\lambda$, $n = 8$.

Case 2: $\underline{a} = 19$ degrees for $\ell = 7\lambda$, $n = 28$.

For Case 1 $\Delta = 12^\circ$ for $\ell = 1.8\lambda$.

For Case 2 $\Delta = 12^\circ$ for $\ell = 4.9\lambda$.

Both cases here suffer from the disadvantage that in order to get a wave angle of 4° the antenna must be worked at its high frequency limit. If the frequency is raised a small percentage the main lobe will disintegrate.

Case 1 suffers from the further disadvantage that the wave angle varies rapidly with frequency of transmission. From a receiving standpoint Case 2 is very sensitive to change in angle of arrival.

Gain Calculations:

	Case 1 Alignment Design	Case 1 Maximum E_Δ Design	Case 2 Alignment Design
Δ (degrees)	4	4	4
$\underline{a}$ (degrees)	35.5	38	19
n	8	8	28
$\frac{\ell}{\lambda}$	2	2	7
λ (meters)	10	10	10
Sin $\underline{a}$	0.581	0.616	0.326
Wire Size	#10AWG	#10AWG	#10AWG
d (inches)	0.102	0.102	0.102
Z_o (ohms)	716-j170	723-j170	647-j170
Z_r (ohms)	710-j40	715-j15	710-j20
f_o^2	0.37	0.373	0.33
f_m^2	0.636	0.635	0.61
Zin(ohms)	716-j195	723-j180	647-j182
T (db)	2.02	1.98	1.75
n^2	64	64	784
$\text{Sin}^2\ \underline{a}$	0.336	0.378	0.106
Cos Δ	0.997	0.997	0.997
Cos $\underline{a}$	0.814	0.788	0.945
β (degrees)	67.7	77.2	74.4
β (radians)	1.18	1.35	1.3
$\text{Sin}^4\beta$	0.729	0.9	0.858
β^2	1.39	1.82	1.69
g_d	18.8	19.8	70.5
G_d (db)	12.7	13	18.5
G_p (db)	16.7	17	22.8
$G_p\ \frac{\lambda}{2}$ (db)	8.5	8.8	14.6

Example 3 -- Optimum design -- variable height

In Example 1 the design was optimum for $\ell = 4\lambda$. The high frequency limit was given by $\ell = 5.5\lambda$ for $\underline{a} = 21$ degrees. This gave $\Delta = 3$ degrees. To maximize the height factor at 3 degrees we must adjust the electrical height H until

$$H = \frac{90}{\sin 3^{\circ}} = 1\,720 \text{ degrees} = 4.78\,\lambda = 29.4 \text{ m}$$

Assuming this is feasible, the 1-2 lobe amplitude would be 0.3, or down 10.6 db from the main lobe. Interference-wise this is still a large lobe. The gain, unaffected by height factor, is 13.8 db. This slight decrease in secondary lobe amplitude has been paid for by the provision of height control. This provision is usually made to take care of changes in propagation conditions over the sunspot cycle.

Example 4 -- Consider the following antenna

$\ell' = 110$ meters, $h/2 = 27.5$ m, $\underline{a} = 22$ degrees, f = 12 mcs,

$\lambda = 25$ m $\qquad \ell = 4.4\lambda$, $h/2 = 1.1\lambda$ No 12AWG wire.

$d = .081$ inches.

The power gain of this antenna with respect to a horizontal dipole will be computed for $\Delta = 0, 5, 10, 15, 20, 25$ and 30 degrees at f = 8 mcs, 12 mcs and 18 mcs. The values so computed will be compared with the semi-experimental values of Christiansen[10] calculated for a horizontal rhombus consisting of a pair of No 12 wires.

The results are tabulated below:

f = 8 mcs

$\lambda = 37.5$ m, $h/2 = 0.733\lambda$, $\frac{I_a}{I_o}$ = ratio of average to input current

$f_m^2 = 0.753$, $f_m = .87$, $\frac{I_a}{I_o} = 0.85$ experimentally

$Z_{in} = 850 - j265$ $\qquad Z_{in} = 700$ ohms experimentally

	Theoretical	Christiansen's results
Δ	$G_{p\frac{\lambda}{2}}$	$G_{p\frac{\lambda}{2}}$
0	6.2 db	7.5 db
5	6.4 db	7.6 db
10	6.9 db	8.5 db
15	7.8 db	8.8 db
20	7.4 db	9 db
25	7.4 db	8.1 db
30	5.2 db	5.5 db

The results here agree quite closely. The experimental results are for a rhombus consisting of a pair of wires. The purpose of this is to lower the surge impedance (Lewin[13]) and hence to increase the gain. Thus if we add 10 log $\frac{850}{700}$ to the single-wire rhombus we get the following figures:

Δ	$G_{p\frac{\lambda}{2}} + 10 \log \frac{850}{700}$	$G_{p\frac{\lambda}{2}}$ (experimental)
0	7.1 db	7.5 db
5	7.3 db	7.6 db
10	7.8 db	8.5 db
15	8.7 db	8.8 db
20	8.3 db	9 db
25	8.3 db	8.1 db
30	6.1 db	5.5 db

For,

$f \quad = 12$ mcs

$\lambda \quad = 25$ m , $\quad h/2 = 1.1\,\lambda$

$f_m^2 = 0.684$, $\quad f_m = .83$, $\quad \dfrac{I_a}{I_o} = 0.80$, experimentally

$Z_{in} = 802 - j122 \quad Z_{in} = 700\ \Omega$ experimentally

Δ	$G_{p\frac{\lambda}{2}}$ (Theoretical)	$G_{p\frac{\lambda}{2}}$ (experimental)
0	11.2 db	11.5 db
5	11.3 db	11.55 db
10	11.4 db	11.5 db
15	10.8 db	10.6 db
20	8.9 db	8.1 db
25	3.4 db	1.2 db
30	< 0	< 0

For,

$f \quad = 18$ mcs, $\quad \ell = 6.6\,\lambda$, $\quad \lambda = 16.67$m , $\quad h/2 = 1.65\,\lambda$

$f_m^2 = 0.635 \quad f_m = .80$, $\quad \dfrac{I_a}{I_o} = 0.77$ experimentally

$Z_{in} = 752 - j180 \quad Z_{in} = 700$ experimentally

Δ	$G_{p\frac{\lambda}{2}}$ (Theoretical)	$G_{p\frac{\lambda}{2}}$ (experimental)
0	14.2 db	13.8 db
5	13.8 db	13.4 db
10	12.3 db	11.5 db
15	7.9 db	6.3 db
20	< 0 db	< 0 db
25	< 0 db	< 0 db
30	< 0 db	< 0 db

Example 5

By assuming all the power is either radiated or dissipated in the termination Christiansen[7] develops the expression

$$R = \frac{I_a^{\,2} - I_T^{\,2}}{I_a^{\,2}} \quad Z_{in}$$

for the radiation resistance where I_a is the average current flowing in the antenna,

I_T is the current in the termination, and

Z_{in} is the input resistance which is equal to the terminating resistance.

I_a, I_T and Z_{in} are all determined experimentally by Christiansen[7].
For the antenna in Example 4 Christiansen[7] obtains:

$R = 570\,\Omega$ at 8 mcs

$R = 760\,\Omega$ at 12 mcs

$R = 810\,\Omega$ at 18 mcs

Using the approximate formula of Lewin[12, 13] one obtains

$R = 540\,\Omega$ at 8 mcs

$R = 640\,\Omega$ at 12 mcs

$R = 735\,\Omega$ at 18 mcs

The real part of Chaney's formula (Chaney[5]) which is identical with Lewin's exact formula; gives by linear interpolation on Figure 7

$R = 512\,\Omega$ at 8 mcs

$R = 654\,\Omega$ at 12 mcs

$R = 742\,\Omega$ at 18 mcs.

Christiansen[7] obtains

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

$$\frac{W_T}{W_o} = 0.41 \text{ at } 8 \text{ mcs}$$

$$= 0.31 \text{ at } 12 \text{ mcs}$$

$$= 0.32 \text{ at } 18 \text{ mcs}$$

Calculating $\frac{W_T}{W_o} = f_o^2$ one obtains the following figures:

$$f_o^2 = 0.57 \text{ at } 8 \text{ mcs}$$

$$= 0.47 \text{ at } 12 \text{ mcs}$$

$$= 0.32 \text{ at } 18 \text{ mcs}$$

Here W_T is the power dissipated in the terminating resistance and W_o is the input power.

VI Conclusions

The method presented in this paper seems to be quite satisfactory and enables one to make a quick first design. If the coordinates and relative amplitudes of the lobes of the pattern are desired these can be quickly obtained using the method presented by Laport[10 11]. The method of this paper can be extended to multi-element rhombic arrays. In this connection the following papers should be referred to:

Chaney, J. G. Simplification for Mutual Impedance of Certain Antennae U. S. Naval Postgraduate School, Technical Report No. 6, November 1952.

Chaney, J. G. Mutual Impedance of Rhombic Antennas Spaced in Tandem. U. S. Naval Postgraduate School, Technical Report No. 7, December 1952.

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